

## Bargaining under incomplete information, fairness, and the hold-up problem

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# Bargaining under Incomplete Information, Fairness, and the Hold-Up Problem

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## Abstract

In the hold-up problem incomplete contracts cause the proceeds of relationship-specific investments to be allocated by bargaining. This paper investigates the corresponding investment incentives if individuals have heterogeneous fairness preferences and thus differ in their bargaining behavior. Individual preferences are taken to be private information. Investments can then signal preferences and thereby influence beliefs and bargaining behavior. In consequence, individuals might choose high investments in order not to signal information that is unfavorable in the ensuing bargaining.

*JEL Classification: L2, C78, D82*

*Keywords: hold-up, relationship-specific investments, fairness, reciprocity, asymmetric information, signalling.*

# 1 Introduction

Over the past decades the study of incomplete contracts has generated numerous insights into the functioning of organizations. Most of this literature is based on the famous hold-up problem as pioneered by Williamson (1975) and formalized by Klein et al. (1978) and Groux (1982). Incomplete contracts cause the proceeds of previously sunk, relationship-specific investments to be allocated by bargaining. Since each trader reaps only part of his investment's proceeds but incurs all costs, investment incentives are inefficiently low. Although many real-world contracts are clearly incomplete, they appear to be quite effective: Professors prepare classes even though their wages do not condition on their teaching record, and customized goods are traded to mutual satisfaction although most trade contracts are silent on many important details. Following Hackett (1994) several articles have studied hold-up situations in controlled laboratory experiments.<sup>1</sup> Supporting the casual empiricism above, they find that investments tend to be higher than expected. Consistent with numerous other bargaining experiments, the authors predominantly propose fairness concerns or social preferences as explanations for their results; but their experiments also suggest that subjects differ widely in their bargaining behavior, where individual preferences seem to be private information.<sup>2</sup> Surprisingly, any possible interactions between fairness preferences and incomplete information have not been fully analyzed thus far.

The present paper studies investment incentives in a hold-up situation if individuals have heterogeneous fairness preferences that are private information. It shows that fairness concerns based on intention-based reciprocity can improve investment incentives, that can even be efficient if fairness preferences are private information.

In the formal analysis a buyer and a seller bargain over the division of a trade surplus that is generated by a relationship-specific investment of the seller. Ex-post bargaining is modeled as an ultimatum game in which the buyer holds all bargaining power. Buyers are assumed to have standard “selfish” preferences, but sellers differ in their preferences and thus in their bargaining behavior: A “selfish” seller accepts all offers that give him a weakly positive share of the trade surplus, whereas a “fair-minded seller” refuses to trade unless he receives a share that exceeds some strictly positive “fair” threshold. Most importantly, the seller's preferences are private information.

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<sup>1</sup>More recent examples are Ellingsen and Johannesson (2004a, 2004b).

<sup>2</sup>See Güth et al. (1990) or Camerer and Thaler (1997) for a summary of the literature on bargaining and social preferences.

Results are as follows. Because the seller's preferences are private information, the ex-post bargaining over the surplus takes place under incomplete information. The buyer's beliefs about the seller's preferences determine his bargaining behavior, but beliefs are updated after observing investments. Incomplete information thus links the otherwise sunk investment to bargaining outcomes: It can be optimal for both selfish and fair-minded sellers to choose a particular investment in order not to signal information on their type that is unfavorable in the ensuing bargaining. Together with incomplete information fairness can thus generate very strong or even efficient investment incentives.

After characterizing all possible equilibrium investments, the paper describes the sufficient and necessary conditions under which there exists an efficient perfect Bayesian equilibrium. On the one hand, the ex-ante probability that the seller is fair-minded must be sufficiently high so that the buyer bargains less aggressively if he does not learn anything about the seller's type. On the other hand, the "fair" share of the trade surplus must be large enough to foreclose profitable deviations. The following is noteworthy. First, the seller might invest efficiently although he does not get the entire trade surplus in equilibrium. Second, when both types of seller invest efficiently, they invest more and therefore do not simply mimic the fair-minded seller under complete information. Third, the seller must be selfish with strictly positive probability as otherwise information is complete and investment incentives are inefficiently low. As in most dynamic games with incomplete information, there are multiple equilibria. The intuitive criterion by Cho and Kreps (1987) is shown to have no bite, but a refinement proposed by Mailath et al. (1993) generates sharp predictions: If the buyer's prior belief is sufficiently high, fairness preferences unambiguously improve investment incentives. However, full efficiency cannot be attained unless the seller's investment choice is discrete.

The present study is most closely related to the following papers on fairness and the hold-up problem. Just as in the current setup, Ellingsen and Johannesson (2004b) consider a situation in which the trade surplus is allocated in an ultimatum game with the buyer as proposer.<sup>3</sup> They show that inequity aversion à la Fehr and Schmidt (1999) then increases equilibrium investments only if many buyers are inequity averse and thus make generous offers. Since in reality rather few subjects are inequity averse, they conclude that inequity aversion has difficulties in explaining the experimental evidence. The present study considers fairness

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<sup>3</sup>Ellingsen and Johannesson (2004a, 2005) differ in their specification of the bargaining process.

preferences that are based on reciprocity.<sup>4</sup> Investment incentives can then be efficient even if all buyers are selfish and the bargaining is modeled as an ultimatum game. Ewerhart (2006) demonstrates that inequity aversion can solve the hold-up problem if the trade surplus is divided in an alternating-offer bargaining game with infinite horizon: If traders are patient and account for investment costs in their comparisons, they essentially divide the net surplus equally so that investment incentives are approximately efficient. However, this result hinges on the bargaining game and thus cannot explain all the experimental evidence.<sup>5</sup> Finally, only the present paper focuses on the signalling incentives that arise if fairness preferences are private information. Tirole (1986) and Gul (2001) argue that investment incentives can improve if investments are private information. Contrary to the present paper, they cannot explain the high investments as documented in experiments with observable investment. Experiments by Hackett (1994) and Sloof et al. (2007) suggest strongly that making investments unobservable does not increase their value.

The remaining paper is organized as follows. Section 2 introduces the model, Section 3 describes all possible equilibrium investments, and Section 4 characterizes the impact of fairness and incomplete information on investment incentives. Section 5 discusses equilibrium refinements, and Section 6 summarizes the main results.

## 2 The Model

A buyer and a seller can trade one unit of a good. The good's quality  $i \in \mathbb{R}_+$  is determined by an investment of the seller. Producing a good of quality  $i$  cost the seller  $i$ . Once these costs are sunk, the absence of a complete contract causes terms of trade to be determined by bargaining. The bargaining process is modeled as an ultimatum game. First, the buyer proposes a share  $p \in \mathbb{R}$  of the trade surplus to be given to the seller. Second, the seller can accept or reject the buyer's offer. Let  $a \in \{0, 1\}$  describe the seller's acceptance decision, where  $a = 1$  if he accepts. If the seller accepts the buyer's proposal  $p$ , then the good is traded, the seller receives share  $p$  of the trade surplus, and the buyer consumes the good. If

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<sup>4</sup>Experiments such as Falk et al. (2008), Charness and Rabin (2002), or Nelson (2002) suggest that intention-based reciprocity strongly influences bargaining behavior.

<sup>5</sup>von Siemens (2004) shows that inequity aversion can also generate efficient investment incentives in an alternating-offer bargaining game with finite horizon if preferences are private information. The present paper studies the same signalling incentives as in von Siemens while referring to preferences that are based on intention-based reciprocity. This simplifies the analysis while offering a better fit to the experimental evidence.

the seller rejects, the good is not traded but consumed by the seller himself.

Experimental investigations of the ultimatum game show the following behavioral patterns. First, some responders reject strictly positive proposals. Second, responders differ as some reject proposals accepted by others. Third, bargaining breakdowns occur frequently so that the minimum proposal a particular responder is willing to accept seems to be private information. A large and growing number of fairness models are consistent with these stylized facts.<sup>6</sup> To sharpen the focus on the importance of incomplete information on fairness preferences, the model does not apply a particular fairness model. Instead, the preferences described below should be considered a reduced form representation of equilibrium utilities that result from fairness preferences based on first principles. Appendix A provides an extensive micro foundation for these preferences in the spirit of intention-based reciprocity à la Rabin (1993) or Dufwenberg and Kirchsteiger (2004).

Let the functions  $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^+$  and  $\psi : \mathbb{R}_+ \rightarrow \mathbb{R}^+$  describe the buyer's and the seller's value from consuming a good of quality  $i$ . The function  $\phi$  is strictly increasing and concave, whereas  $\psi$  is weakly increasing and concave. Both  $\phi$  and  $\psi$  are twice differentiable, where in addition  $\lim_{i \rightarrow 0} \phi'(i) = +\infty$  and  $\lim_{i \rightarrow \infty} \phi'(i) = 0$  to ensure inner solutions. Finally,  $\phi(0) = \psi(0) = 0$  and  $\phi'(i) > \psi'(i)$  for all  $i \in \mathbb{R}_+$ . The trade surplus  $\phi(i) - \psi(i)$  is thus strictly increasing and strictly positive for all but the minimum investment.

Preferences can now be defined in terms of investment costs and shares of the trade surplus. Depending on the good's quality  $i$ , the buyer's proposal  $p$ , and the seller's acceptance decision  $a$ , the buyer's payoff is defined as

$$u_b(i, p, a) = a(1 - p)[\phi(i) - \psi(i)]. \quad (1)$$

If the good is not traded, he receives a payoff normalized at zero. The buyer is thus taken to be exclusively interested in his share of the trade surplus. The impact of fairness preferences on the side of the buyer is briefly examined at the end of Section 4.

The seller can be either selfish or fair-minded. Let  $\theta \in \{s, f\}$  denote the seller's type, where  $\theta = s$  indicates that the seller is selfish and  $\theta = f$  indicates that he is fair-minded. The

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<sup>6</sup>The literature can be roughly divided into three strands: Distributional fairness concerns as in Fehr and Schmidt (1999) or Bolton and Ockenfels (2000), intention-based reciprocity as in Rabin (1993) or Dufwenberg and Kirchsteiger (2004), and a mixture of both as in Falk and Fischbacher (2006).

seller's type is taken to be private information. It is common knowledge that the ex-ante probability that the seller is fair-minded is  $\pi \in [0, 1]$ . Depending on the good's quality  $i$ , the proposal  $p$ , and the acceptance decision  $a$ ,

$$u_s(i, p, a) = a p [\phi(i) - \psi(i)] + \psi(i) - i \quad (2)$$

describes a selfish seller's payoff. Let

$$u_f(i, p, a) = \begin{cases} u_s(i, p, a) & \text{if } p \geq \gamma \\ u_s(i, p, a) - a K & \text{if } p < \gamma \end{cases} \quad (3)$$

denote a fair-minded seller's payoff for some strictly positive parameter  $\gamma \in ]0, 1[$ . This can be interpreted as follows: A fair-minded seller considers proposals below some threshold  $\gamma$  as unfair. He incurs costs  $K$  if he accepts, but he avoids these behavioral costs by retaliating and rejecting the offer. Note that a fair-minded seller's utility is discontinuous at  $p = \gamma$ . The importance of this discontinuity is discussed at the end of Section 4.

In the present paper the fair share  $\gamma$  is taken to be constant. In contrast to the existing literature adding fairness preferences to the hold-up problem, there is no direct link between investments and the evaluation of bargaining outcomes. In reality sunk costs probably affect bargaining behavior by changing what is considered a fair allocation of the surplus.<sup>7</sup> The present paper studies the connection between investment incentives and bargaining outcomes created by incomplete information. To clarify the analysis it excludes all confounding effects of fairness preferences. The following assumption further simplifies the analysis.

**Assumption 1**  $\gamma [\phi(i) - \psi(i)] < K$  for all  $i$ .

A fair-minded seller's behavioral costs from accepting unfair offers are therefore high.<sup>8</sup> As will become clear in the ensuing analysis, his acceptance decision thus depends only on whether the offered share of the trade surplus exceeds the threshold  $\gamma$ . Without incomplete information, sunk costs are thus sunk; they have no impact on bargaining outcomes in any way other than by influencing the buyer's belief.

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<sup>7</sup>See in particular the empirical evidence in Ganter et al. (2001).

<sup>8</sup>Assumption 1 requires that the trade surplus is bounded above. Note that this need not conflict with the monotonicity assumption  $\phi'(i) > \psi'(i)$  for all  $i$ .



Since the trade surplus  $\phi(i) - \psi(i)$  is strictly positive for all investment levels, the good should always be traded. By definition let the efficient investment

$$i_e = \arg \max_{i \in \mathbb{R}} \{\phi(i) - i\} \quad (4)$$

maximize the pie  $\phi(i) - i$  that can be distributed between buyer and seller. Moreover, even if there is no trade, the seller can consume the good himself. He then gets  $\psi(i) - i$ . Optimizing over his investment, his maximum outside option is

$$U_0 = \max_{i \in \mathbb{R}} \{\psi(i) - i\}. \quad (5)$$

Let  $i_0$  denote the corresponding maximizer.

### Strategies and Perfect Bayesian Equilibria

Since preferences are private information, the hold-up problem now forms a dynamic game of incomplete information. The corresponding solution concept is perfect Bayesian equilibrium. Since the seller's investment can signal his type, let  $\mu : \mathbb{R}_+ \rightarrow [0, 1]$  denote the probability with which the buyer believes the seller to be fair-minded after observing investment  $i$ . Mixed strategies will be considered, but the paper primarily focuses on pure strategies since efficiency requires that both types of seller always choose the efficient investment. A pure strategy for the buyer determines a proposal  $p_b : \mathbb{R}_+ \rightarrow \mathbb{R}$  conditional on the observed investment  $i$ . A pure strategy for the seller determines an investment  $i_\theta$  and an acceptance rule  $a_\theta : \mathbb{R} \rightarrow \{0, 1\}$  for each type  $\theta \in \{f, s\}$ . The acceptance rule describes his acceptance decision conditional on the buyer's proposal  $p$ . Remember that a fair-minded seller's acceptance decisions depend on only whether the proposal exceeds  $\gamma$ . As the latter is independent of the investment, the fair-minded seller's acceptance decision need not condition on the investment. Following conventions in contract theory, a seller is taken to accept a proposal whenever he is indifferent between accepting and rejecting.

Let  $\mathcal{C}(i, \mu)$  denote an ultimatum game characterized by an investment  $i$  of the seller and a belief  $\mu$  of the buyer. A perfect Bayesian equilibrium (PBE) then has the following properties. First, in every ultimatum game  $\mathcal{C}(i, \mu^*(i))$ , the buyer's equilibrium strategy  $p_b^*$  and the seller's equilibrium acceptance rule  $a_\theta^*$  are mutually optimal given the equilibrium belief  $\mu^*(i)$ . Acceptance rules therefore comply with the spirit of subgame-perfection. Second, whenever possible the buyer's belief  $\mu^*$  is formed according to Bayes' rule. If an investment is never chosen in equilibrium, Bayes' rule is not applicable and the buyer's belief may be set freely.

Finally, each type of the seller's equilibrium investment  $i_\theta^*$  is optimal given the anticipated equilibrium play in the ultimatum games  $\mathcal{C}(i, \mu^*(i))$ .

### 3 Description of Perfect Bayesian Equilibria

Perfect Bayesian equilibria and the corresponding investment incentives are determined by backwards induction. Consider an ultimatum game  $\mathcal{C}(i, \mu)$  characterized by investment  $i$  and belief  $\mu$  of the buyer. Since the trade surplus is always positive, a selfish seller optimally accepts a proposal  $p$  if and only if  $p \geq 0$ . By Assumption 1 a fair-minded seller optimally accepts a proposal  $p$  if and only if  $p \geq \gamma$ . The buyer accounts for the bargaining behavior of the seller in the following way. The simple formal proof is omitted.

**Lemma 1 (Equilibrium Bargaining)** *Consider any PBE in any ultimatum game  $\mathcal{C}(i, \mu)$ .*

1. *If  $\mu > \gamma$ , then the buyer proposes  $p = \gamma$  and both types of sellers accept. The sellers get a payoff of  $\gamma[\phi(i) - \psi(i)] + \psi(i) - i$  while the buyer gets a payoff of  $(1 - \gamma)[\phi(i) - \psi(i)]$ .*
2. *If  $\mu < \gamma$ , then the buyer proposes  $p = 0$  and a selfish seller accepts while a fair-minded seller rejects. Both types of sellers get a payoff of  $\psi(i) - i$  while the buyer gets a payoff of  $(1 - \mu)[\phi(i) - \psi(i)]$ .*
3. *If  $\mu = \gamma$ , then the buyer mixes with any probability between the above alternatives with the corresponding acceptance decisions and payoffs.*

The buyer essentially faces two alternatives: He either claims only part of the trade surplus and then trades with certainty, or he claims the entire trade surplus and then trades if and only if the seller is selfish and thus only with probability  $\mu$ . The buyer's belief  $\mu$  determines which alternative is optimal.

#### Pooling Equilibria

Given equilibrium behavior in each ultimatum game, the PBE of the overall game can be derived. The main result of the paper is the derivation of the sufficient and necessary conditions under which there exists an efficient PBE. In the efficient equilibrium both types of seller always choose the efficient, and thus the same, investment. It therefore forms a pooling equilibrium. Pooling equilibria consequently play a special role in the ensuing analysis.

In a pooling equilibrium both types of seller choose the same equilibrium investment. They can deviate only to out-of-equilibrium investments. Since Bayes rule is then not applicable, the buyer's belief can be chosen freely. By Lemma 1 it is most disadvantageous for the seller if the buyer believes him to be selfish. Maximizing with respect to his investment, the maximum outside option  $U_0$  is the lowest maximum deviation payoff the seller can get in any PBE. Thus, an investment can be an equilibrium choice in a pooling equilibrium if and only if both types get an equilibrium payoff that exceeds  $U_0$ . Let  $I$  be the set of investments that (given the buyer proposes  $\gamma$ ) yield the seller a payoff weakly higher than his outside option  $U_0$ . Concavity of  $\phi$  and  $\psi$  directly imply the following lemma.

**Lemma 2** *For all  $\gamma \in ]0, 1[$  the set  $I = \{i \in \mathbb{R}_+ : \gamma[\phi(i) - \psi(i)] + \psi(i) - i \geq U_0\}$  forms a non-empty, bounded, and closed interval  $[\underline{i}, \bar{i}] \in \mathbb{R}_+$ .*

The set of pooling equilibria is given by the following proposition. Formal proofs for the ensuing results can be found in Appendix B.

**Proposition 1 (Pooling Equilibria)**

1. *If  $\pi \geq \gamma$ , then there exists a continuum of pooling equilibria where both types of sellers invest  $i^* \in I$  and the good is always traded.*
2. *If  $\pi < \gamma$ , then there exists a unique pooling equilibrium in which both types of sellers invest  $i_0$  and the good is traded if and only if the seller is selfish.*

The intuition for this result is illustrated in Figure 1. If the seller's type is private information, his payoff depends on the buyer's belief. In a pooling equilibrium the buyer does not learn anything after observing the equilibrium investment. If the buyer's prior belief is sufficiently high, he then offers share  $\gamma$  of the trade surplus. Depending on the equilibrium investment both types of sellers get an equilibrium payoff of  $U_\theta^*$  lying on the upper curve. Observing a deviation, the buyer believes the seller to be selfish and claims the entire trade surplus. By choosing the right deviation both types of sellers can move along the lower curve. They can therefore get a deviation payoff of up to  $U_0$ . Comparing equilibrium and deviation payoffs, all investments in  $[\underline{i}, \bar{i}]$  can be equilibrium investments in a pooling equilibrium.

**Separating and Semi-Separating Equilibria**

Although the paper concentrates on the existence of the efficient pooling equilibrium, this section characterizes the set of all investments that can be supported in any PBE. For once,

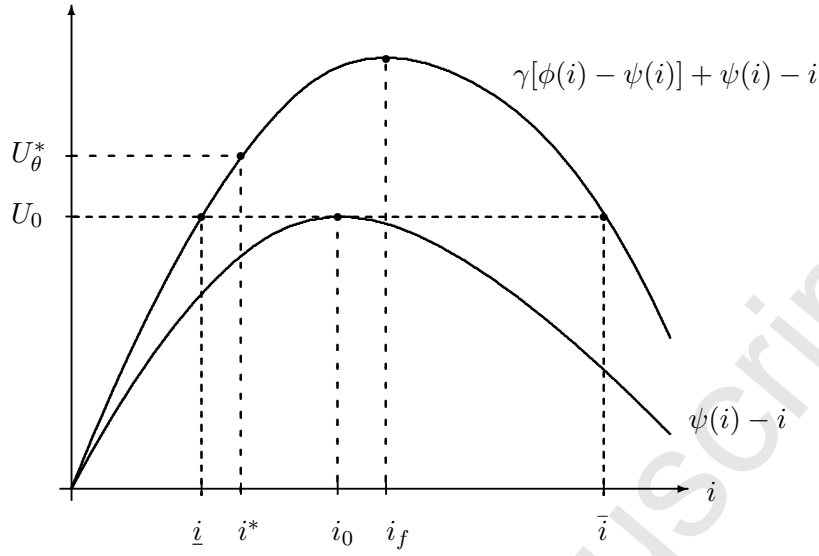


Figure 1: The impact of incomplete information for the case  $\pi \geq \gamma$ .

there exist pure-strategy separating equilibria in which fair-minded sellers choose investments  $i^* \in \{\underline{i}, \bar{i}\}$  while selfish sellers invest  $i_0$ . Buyers propose  $p_b^*(i^*) = \gamma$  and  $p_b^*(i) = 0$  for all  $i \neq i^*$  and the good is always traded in equilibrium. In these separating equilibria selfish sellers do not imitate fair-minded sellers: Although they then get a higher share of the trade surplus, the trade surplus is small or they have to invest a lot and thus incur high costs. Fair-minded sellers do not want to mimic selfish sellers since the buyer then claims the entire surplus. There also exist mixed-strategy equilibria. The set of any possible equilibrium investments in any PBE can be characterized as follows.

**Proposition 2 (Equilibrium Investments)** *There exists a PBE in which investment  $i^*$  is chosen with strictly positive probability if and only if  $i^* \in I$ .*

This result has the following intuition. First, it can never be optimal to choose an investment outside the set  $I$ . The reason is that the buyer never makes equilibrium proposals exceeding  $\gamma$ . Both types of sellers thus get an equilibrium payoff strictly lower than  $U_0$  if they invest  $i \notin I$ . These investments are strictly dominated by investing  $i_0$ , which yields the seller at least his outside option  $U_0$ . Second, for every investment  $i \in I$  one can find a PBE in which this investment is chosen with strictly positive probability. By Proposition 1 there exists such a pooling equilibrium if the buyer's prior  $\pi$  is sufficiently high. Otherwise, it is possible to construct a semi-separating equilibrium with the desired properties. Appendix B provides a full characterization of these equilibria.

## 4 Incomplete Information and Investment Incentives

This section first disentangles the impact of fairness and incomplete information by investigating the benchmark case of complete information. Suppose the seller's preferences are observable. As the buyer knows the seller's type, his bargaining behavior is independent of the seller's investment. Investing  $i$  a selfish seller receives a payoff of  $\psi(i) - i$  whereas a fair-minded seller gets a payoff of  $\gamma[\phi(i) - \psi(i)] + \psi(i) - i$ . Graphically speaking, a fair-minded seller can move along the upper curve whereas a selfish seller can move along the lower curve in Figure 1. Equilibrium investments maximize payoffs, which implies the following proposition. The simple formal proof is omitted.

**Proposition 3 (Complete Information Equilibrium )** *Suppose the seller's type is observable. Then there exists a unique subgame-perfect Nash equilibrium in which the good is always traded, the selfish seller chooses  $i_0$ , and the fair-minded seller chooses*

$$i_f = \arg \max_i \{ \gamma [\phi(i) - \psi(i)] + \psi(i) - i \}.$$

*Since  $i_0 < i_f < i_e$  both types of sellers under-invest.*

The buyer bargains less aggressively if and only if the seller is fair-minded. Since a fair-minded seller receives a larger share of the trade surplus, the assumptions on  $\phi$  and  $\psi$  imply that he invests more than a selfish seller, but since not even a fair-minded seller receives all the trade surplus, his equilibrium investment is inefficiently low. Moreover, the selfish seller's investment incentives are not improved at all.

If the seller's type is private information, the buyer's bargaining behavior depends on his belief. The buyer's beliefs might be influenced by the seller's investment. The resulting signalling can have the following consequences. First, incentives for both types might be improved. For example, in a pooling equilibrium the buyer's reaction to the equilibrium investment is independent of the seller's type. If the prior probability for the seller to be fair-minded is sufficiently high, the buyer bargains as if the seller were fair-minded even if the seller is in fact selfish. The investment incentives of both types of sellers are thus affected. Second, the change in the buyer's belief can affect his equilibrium proposal and therefore the seller's equilibrium payoff in a discontinuous way. Consider a pooling equilibrium in which the buyer makes the generous proposal  $\gamma$  when he has learned nothing about the seller's type. Given that the buyer believes the seller to be selfish when observing a deviation, a seller gets share  $\gamma$  of the trade surplus if and only if he chooses the equilibrium investment. If he invests

marginally less, the buyer's belief changes. This affects the buyer's proposal, which causes the seller's payoff to drop dramatically. Incomplete information can therefore generate very strong incentives to make a particular investment.

### Existence of Efficient Perfect Bayesian Equilibrium

If there is incomplete information, there can indeed exist an equilibrium in which both types of seller invest efficiently and the good is always traded. By Proposition 1 such an efficient equilibrium exists if and only if the buyer's prior belief is sufficiently high and the efficient investment  $i_e$  lies in the interval  $I$ . Consequently, inequality  $\gamma[\phi(i_e) - \psi(i_e)] + \psi(i_e) - i_e \geq U_0$  must hold. Solving for  $\gamma$  yields the following corollary.

**Corollary 1 (Efficient Equilibrium)** *There exists an efficient PBE if and only if*

$$\gamma \geq \frac{(\psi(i_0) - i_0) - (\psi(i_e) - i_e)}{\phi(i_e) - \psi(i_e)}$$

*and the buyer's prior belief satisfies  $\pi \geq \gamma$ .*

This condition has the following interpretation. As the seller can reject the buyer's proposal, he always gets at least his outside option. He can thus secure himself his maximum outside option  $\psi(i_0) - i_0$  by investing  $i_0$ . If he invests efficiently, his outside option drops to  $\psi(i_e) - i_e$ . This reduces his incentives to invest efficiently. However, the seller then gets share  $\gamma$  of the efficient trade surplus  $\phi(i_e) - \psi(i_e)$  in addition to his outside option. An efficient equilibrium thus exists in case the efficient trade surplus is large as compared to the reduction in the seller's outside option. In other words, an efficient equilibrium can arise if gains from trade are large while the seller's investment has no great impact on his outside option.

Note first that incomplete information does not simply induce all types of sellers to mimic the fair-minded seller's behavior under complete information; in the efficient equilibrium both selfish and fair-minded sellers invest more than the inefficiently low investment  $i_f$  chosen by the fair-minded seller if preferences are observable. Second, the cutoff for  $\gamma$  in Corollary 1 is strictly smaller than unity by the definition of  $i_0$  and  $i_e$ . In equilibrium both types of seller invest efficiently even though they know that they will not receive the entire trade surplus. This is optimal as the buyer's belief changes so that he gets a much smaller share of the trade surplus whenever he invests less. Finally, the buyer's prior must be sufficiently large. As in many models with incomplete information there is a discontinuity: Investment incentives can be efficient only if sellers are not all fair-minded. Otherwise, there is complete information, and the inefficiently low  $i_f$  is the unique equilibrium investment.

### Inefficient Perfect Bayesian Equilibria

The previous section has characterized conditions for the existence of an efficient PBE, but fairness and incomplete information need not result in efficiency. First, Proposition 1 and the semi-separating equilibria characterized in Appendix B demonstrate that the good need not always be traded in equilibrium. The ex-post allocation of the good can thus be inefficient. This connects the analysis to recent studies which, following Williamson (2000), emphasize that ex-post inefficiencies are an important ingredient of transaction costs economics. For example, Ellingsen and Johannesson (2005) conduct an experiment in which investment costs are private information. Social preferences and uncertainty concerning the fairness of a bargaining proposal can then cause bargaining breakdowns. Hart and Moore (2008) argue that contracts set reference points. This promotes ex-post “shading” and can thereby cause inefficiencies even in the absence of relationship-specific investments.

In addition, the present analysis shows that social preferences and incomplete information can strongly distort investments even if ex-post bargaining is efficient in equilibrium. If the selfish seller’s complete information investment  $i_0$  is strictly positive, there can exist PBE in which both types of sellers invest less than  $i_0$ . Figure 1 provides an example for such an equilibrium; although the trade surplus increases when the seller invests more, his share of the trade surplus drops. This makes higher investments unprofitable. By the same reasoning both types might overinvest in equilibrium.

### Discontinuous Preferences, Inequity Aversion, and Fair-Minded Buyers

Extending the argument in Ellingsen and Johannesson (2004b) to continuous investment choices, the following reasoning shows that the assumed discontinuity in the fairness preferences is crucial for investment incentives. If the fair-minded seller’s preferences are continuous as in the model of inequity aversion by Fehr and Schmidt (1999), then there exists a share  $\tilde{p}$  of the trade surplus that makes the fair-minded seller indifferent between accepting and rejecting. If the buyer believes the seller to be fair-minded, he proposes  $\tilde{p}$ . A fair-minded seller accepts and receives a payoff equal to his outside option. If the buyer believes the seller to be selfish, he claims the entire trade surplus. A fair-minded seller rejects the proposal and gets his outside option. A fair-minded seller thus always receives his outside option independently of the buyer’s belief and the corresponding proposal. His equilibrium investment maximizes his outside option.<sup>9</sup>

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<sup>9</sup>This holds true no matter whether traders incorporate investment costs in their social comparisons or not.

Ellingsen and Johannesson (2004b) conclude that inequity aversion can improve incentives only if many buyers suffer from favorable inequity and make equitable offers. Because there are relatively few inequity averse subjects, inequity aversion has difficulties explaining the experimental evidence. In the present setting, fairness concerns of the buyer might actually have a negative impact on incentives. Suppose there are two types of buyer, a selfish buyer with preferences as previously assumed, and a fair-minded buyer who always offers the seller the share  $\gamma$  of the trade surplus, no matter whether he believes the seller to be fair-minded or selfish. Let the buyer's type be private information, where  $\pi_b$  denotes the prior probability for the buyer to be fair-minded. Even if the seller appears to be selfish he thus receives share  $\gamma$  of the trade surplus with probability  $\pi_b$ . The seller's maximum deviation payoff then increases to  $U_0 = \max_i \{ \pi_b \gamma [\phi(i) - \psi(i)] + \psi(i) - i \}$ . Since the seller's equilibrium share remains  $\gamma$  if the buyer believes him to be fair-minded, the set  $I$  of equilibrium investments shrinks. Fairness concerns of the buyer thus make the efficient pooling equilibrium harder to sustain.

## 5 Refinements and Equilibrium Selection

Since there are multiple perfect Bayesian equilibria, the impact of incomplete information on the efficiency of incomplete contracts depends on equilibrium selection. This section studies whether equilibrium refinements can sharpen model predictions.

### Equilibrium Dominance and Intuitive Criterion

Consider first the concept of Equilibrium Dominance, which equals the Intuitive Criterion by Cho and Kreps (1987) if there are only two types. Let a buyer's best response be a proposal which is optimal for some belief. A mixed best response is a mixed strategy over the set of best responses. In the present setting the buyer's best responses are  $\gamma$  and zero, and a mixed best response is a randomization over these two proposals. Let a deviation investment be equilibrium dominated for a certain type of seller if that type gets less than his equilibrium payoff even if the buyer makes the most favorable best response and that type of seller then takes the optimal acceptance decision.

According to Equilibrium Dominance the buyer must not believe the seller to be of a certain type if the out-of-equilibrium investment is equilibrium dominated for this, but not for the other type. However, in the present model both types of sellers get exactly the same payoff for any equilibrium response of the buyer: If the buyer proposes  $\gamma$ , both accept and get a



payoff of  $\gamma[\phi(i) - \psi(i)] + \psi(i) - i$ . If the buyer proposes zero, the selfish seller accepts or rejects whereas the fair-minded seller always rejects. Either type then gets a payoff of  $\psi(i) - i$ . Equilibrium and maximum deviation payoffs are therefore identical for the selfish and the fair-minded seller. Since a deviation cannot be equilibrium dominated for one, but not for the other type, Equilibrium Dominance has no bite.<sup>10</sup>

### Undefeated Equilibrium

Consider next the concept of Undefeated Equilibrium as proposed by Mailath et al. (1993). They restrict out-of-equilibrium beliefs as follows. Take a PBE in which investment  $i$  is never chosen in equilibrium. Suppose that there exists another PBE in which investment  $i$  is chosen in equilibrium by some types and that exactly these types get a higher equilibrium payoff in the second than in the first equilibrium. Out-of-equilibrium beliefs in the first PBE must then satisfy the following conditions. First, the buyer believes sellers to have chosen  $i$  with probability one if their type is strictly better off in the second than in the first equilibrium. Second, the buyer may believe sellers to have chosen  $i$  with any probability if their type gets the same equilibrium payoff in both the equilibria considered. Finally, the buyer does not believe sellers to have chosen  $i$  if their type never chooses  $i$  in the second equilibrium.

This refinement greatly reduces the set of plausible pure-strategy PBE. Suppose the buyer holds a high prior  $\pi > \gamma$  so that there exists a continuum of pooling equilibria. Take any pooling equilibrium in which sellers do not invest  $i_f$ . Consider a deviation to investment  $i_f$ . There then exists a pooling equilibrium in which both types of seller get  $\gamma[\phi(i_f) - \psi(i_f)] + \psi(i_f) - i_f$ . Because this is the maximum equilibrium payoff, both types of seller receive strictly more in this than in the current equilibrium. The refinement then requires the buyer to hold his prior belief when observing the out-of-equilibrium investment  $i_f$ . As his prior is high, he offers the fair share of the trade surplus, and sellers have an incentive to deviate to  $i_f$ .

This logic eliminates the separating equilibria and all pooling equilibria except the pooling equilibrium in which both types of seller invest  $i_f$ . Next suppose the buyer holds a low prior belief  $\pi < \gamma$ . There then exist the separating equilibria and a unique pooling equilibrium in which both types of seller invest  $i_0$  and the buyer claims the entire trade surplus. The

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<sup>10</sup>By the same argument the more advanced criterion D1 developed by Cho and Kreps (1987) has no effect. The set of mixed best responses yielding exactly as much or strictly more than the equilibrium payoff are identical for both types of sellers.

good is traded if and only if the seller is selfish. Finally, the refinement has no bite in the knife-edge case  $\pi = \gamma$ . The buyer can then optimally claim the entire trade surplus even if he keeps his prior belief. This equilibrium response makes deviations unprofitable, and all investments  $i \in \mathcal{I}$  can be supported as equilibrium choice in a pooling equilibrium.

### **Fairness, Efficiency, and the Empirical Evidence**

The impact of incomplete information thus depends on the applied equilibrium refinement. Equilibrium Dominance and the Intuitive Criterion have no bite. If the efficient equilibrium exists, incomplete information can have a positive impact on investment incentives. But there also exist highly inefficient equilibria. Undeclared Equilibrium strongly reduces the set of plausible pure-strategy equilibria. If the buyer holds a high prior belief, incomplete information unambiguously increases efficiency as both types of seller now choose the high investment  $i_f$ , but efficiency cannot be attained as the efficient equilibrium is eliminated by the refinement.

The latter predictions appear to be consistent with the experimental evidence. In Ellingsen and Johannesson (2004b) sellers face a binary effort choice where making the investment is efficient.<sup>11</sup> Although there should be zero equilibrium investment without fairness concerns, the observed investment rate in different treatments varies between 35% to 64%, high but clearly not fully efficient. The reason might be that some buyers bargain tough, and indeed bargaining breaks down in 6% to 21% of the cases. This suggests that incomplete information on the seller's acceptance behavior plays an important role in the bargaining.

## **6 Summary**

The present paper explores whether fairness preferences and incomplete information can improve investment incentives in a hold-up problem. If the seller's preferences are private information, investments might affect the buyer's beliefs about the seller's type. Since the buyer's beliefs influence his bargaining behavior, incomplete information links the otherwise sunk investment costs to bargaining outcomes. This can generate very strong incentives to choose a particular investment. There are multiple equilibria and investment incentives need not be efficient, but an equilibrium refinement as proposed by Mailath et al. (1993)

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<sup>11</sup>Small deviations from the efficient investment are not feasible with binary investments. Given the current preferences all sellers investing can form an undeclared equilibrium if the buyers' prior beliefs are high.

considerably sharpens model predictions. In particular, fairness concerns unambiguously improve investment incentives if the prior probability with which a seller is fair-minded is sufficiently high.

## Appendix A: Intention-Based Reciprocity

This section shows that equilibrium behavior and equilibrium utilities as characterized in Lemma 1 can be considered as a reduced form description of a fairness equilibrium built on intention-based reciprocity à la Rabin (1993) or Dufwenberg and Kirchsteiger (2004).

### Material Payoffs and Kindness

In Rabin (1993) a fair-minded trader's utility depends on his and the other trader's kindness. These in turn depend on the material payoffs the traders expect to yield each other. Given investment  $i$ , offer  $p$ , and acceptance decision  $a_\theta$ , let  $m_b(i, p, a_\theta) = a_\theta (1 - p)[\phi(i) - \psi(i)]$  be the material payoff of a buyer, and let  $m_\theta(i, p, a_\theta) = \psi(i) - i + a_\theta p[\phi(i) - \psi(i)]$  be the material payoff of a seller of type  $\theta \in \{s, f\}$ .

Take a seller of type  $\theta \in \{s, f\}$  who follows acceptance rule  $a_\theta$ . Let  $m_{\theta l}(i, a_\theta)$  and  $m_{\theta h}(i, a_\theta)$  denote his minimum and maximum monetary payoff given that a buyer offers some  $p \leq 1$ .<sup>12</sup> By definition a seller's fair material payoff is  $\gamma [\phi(i) - \psi(i)] + \psi(i) - i$ . Then

$$f_b(i, p, a_\theta) = [m_\theta(i, p, a_\theta) - [\gamma [\phi(i) - \psi(i)] + \psi(i) - i]] / [m_{\theta h}(i, a_\theta) - m_{\theta l}(i, a_\theta)] \quad (6)$$

is the kindness of a buyer who offers  $p$  while facing a seller with acceptance rule  $a_\theta$ . Given the seller's acceptance rule  $a_\theta$  a buyer's kindness depends on the material payoff  $m_\theta(i, p, a_\theta)$  he gives the seller relative to the fair material payoff. It is normalized by the maximum payoff variation  $m_{\theta h}(i, a_\theta) - m_{\theta l}(i, a_\theta)$ . If the latter is zero, the buyer cannot affect the seller's material payoff. In this case his kindness is set to zero.

Analogously to the above let  $m_{bl}(i, p)$  and  $m_{bh}(i, p)$  be the minimum and maximum material payoff of a buyer who offers share  $p$  given that the seller either accepts or rejects. A buyer's

<sup>12</sup>As will become clear, the buyer's kindness depends on offers he could but did not make. The buyer could always be very kind and offer the seller more than the trade surplus, yet he then gets less than his outside option. Consequently, the buyer never offers more than the entire trade surplus in any equilibrium. Anticipating this, offers that are generous but unrealistic should not affect the perceived kindness of the buyer. See also Rabin (1993, p.1292, footnote 18).

fair material payoff is  $(1 - \gamma)[\phi(i) - \psi(i)]$ . Then

$$f_\theta(i, p, a) = [m_b(i, p, a) - (1 - \gamma)[\phi(i) - \psi(i)]] / [m_{bh}(i, p) - m_{bl}(i, p)] \quad (7)$$

is the kindness of a seller of type  $\theta \in \{s, f\}$  who takes acceptance decision  $a \in \{0, 1\}$  while facing offer  $p$  after investing  $i$ . If the denominator is zero, his kindness is set to zero.

### Beliefs, Higher Order Beliefs, and Preferences

A priori a buyer does not know the acceptance rules of sellers, so he must form beliefs to assess his own kindness. Further, to assess the kindness of a buyer's offer, a seller must form beliefs concerning the buyer's beliefs. The importance of such beliefs and higher order beliefs formalizes intentions. It is the essence of Rabin (1993) or Dufwenberg and Kirchsteiger (2004).

Further, the present paper must extend their equilibrium concepts to account for incomplete information. It is not obvious how to assess the kindness of a buyer who does not know the seller's type and thus the latter's acceptance rule, but expected kindness appears to be a natural starting point. Thus, suppose the seller believes the buyer believes the seller to follow acceptance rules  $\tilde{a}_\theta$ . Further, suppose the seller believes the buyer believes the seller to be fair-minded with probability  $\tilde{\mu}$ . Then

$$E_\theta f_b(i, p, \tilde{a}_\theta) = \tilde{\mu} f_b(i, p, \tilde{a}_f) + (1 - \tilde{\mu}) f_b(i, p, \tilde{a}_s) \quad (8)$$

is the perceived kindness of a buyer offering  $p$ .

Due to the sequential structure of the game (the buyer takes no actions after the seller's acceptance decision), beliefs and higher order beliefs are irrelevant for the self-assessment of the seller's kindness. A fair-minded seller's utility is then defined as

$$u_f(i, p, \tilde{a}_\theta, \tilde{\mu}, a) = m_f(i, p, a) + K E_\theta f_b(i, p, \tilde{a}_\theta) [1 + f_f(i, p, a)] \quad (9)$$

where  $K \in \mathbb{R}_+$  measures the importance of the seller's reciprocity. In case the seller thinks the buyer is kind, he wants to reciprocate by being kind himself. Formally, if  $E_\theta f_b(i, p, \tilde{a}_\theta) > 0$  then a fair-minded seller's utility is strictly increasing in  $f_\theta(i, p, a)$ . Analogously, a fair-minded seller wants to be unkind to reciprocate unkind behavior of the buyer. Finally, selfish sellers and buyers have no fairness concerns. Their utility equals their monetary payoffs.

### Fairness Equilibrium

In a fairness equilibrium (or perhaps a Bayesian fairness equilibrium), offers and acceptance rules are individually rational given beliefs, and beliefs are consistent with the equilibrium strategies and the prior beliefs concerning the seller's type. The following lemma shows that acceptance rules as in Lemma 1 can result from a fairness equilibrium if fairness concerns are sufficiently strong.

**Lemma 3 (Reciprocal Acceptance Decisions)** *Consider an ultimatum game  $\mathcal{C}(i, \mu)$ . Then for  $K$  sufficiently large there exists a fairness equilibrium in which*

$$a_s^*(i, p) = \begin{cases} 1 & \text{if } p \geq 0 \\ 0 & \text{if } p < 0 \end{cases} \quad \text{and} \quad a_f^*(i, p) = \begin{cases} 1 & \text{if } p \geq \gamma \\ 0 & \text{if } p < \gamma \end{cases} \quad (10)$$

*describe the seller's equilibrium acceptance rules  $a_\theta^*$ .*

**Proof:** Since he only cares for his monetary payoff, a selfish seller optimally accepts if and only if  $p \geq 0$ . In the following consider a fair-minded seller. It is optimal for a fair-minded seller to accept offer  $p \in [0, 1]$  if and only if

$$m_f(i, p, 1) - m_f(i, p, 0) + K E_\theta f_b(i, p, \tilde{a}_\theta) [f_f(i, p, 1) - f_f(i, p, 0)] \geq 0. \quad (11)$$

Note that  $f_f(i, p, 1) - f_f(i, p, 0) = 1$  for all  $p \leq 1$  whereas  $m_{\theta h}(i, a_\theta) - m_{\theta l}(i, a_\theta) = \phi(i) - \psi(i)$  for all  $\theta \in \{s, f\}$  given  $a_\theta^*$  as in (10). There are three cases. First, suppose  $p \geq \gamma$ . The buyer correctly anticipates that selfish and fair-minded sellers accept so that  $E_\theta f_b(i, p, \tilde{a}_\theta) = p - \gamma$ . Since this expression is positive, the buyer is kind and a fair-minded seller's material interests and fairness concerns are aligned. He optimally accepts. Second, suppose  $p \in ]0, \gamma[$ . The buyer correctly anticipates that only selfish sellers accept so that  $E_\theta f_b(i, p, \tilde{a}_\theta) = p(1 - \mu) - \gamma < 0$ . Since the buyer is unkind, a fair-minded seller faces a conflict between his material interests (he should accept) and his fairness concerns (he should reject). Rejecting is optimal if and only if his fairness concerns are sufficiently strong or  $K \geq [p[\phi(i) - \psi(i)]]/[\gamma - (1 - \mu)p]$ . Finally, suppose  $p \leq 0$ . By the above argument the buyer is unkind while it is no longer in the material interest of a seller to accept the offer. He optimally rejects. Therefore, strategy  $a_\theta^*$  as specified in (10) is individually rational for  $K$  sufficiently large. Q.E.D.

Maximizing the buyer's expected utility given the acceptance rules in Lemma 3 generates equilibrium utilities that are qualitatively the same as in Lemma 1. First, if the buyer rather believes the seller to be fair-minded, he offers him the fair share  $\gamma$  of the trade surplus. As

this is neither kind nor unkind, the reciprocity term in a fair-minded seller's utility function is zero. Both types of seller accept and get the same equilibrium utility.

Second, if the buyer believes the seller to be selfish, he claims the entire trade surplus. A selfish seller accepts whereas a fair-minded seller rejects this offer. In this case a fair-minded seller gets a strictly lower equilibrium utility than a selfish seller. This has no consequences for the ensuing analysis; the utility difference  $K\gamma^2$  is constant so that both types of seller choose the same investment if they expect to get only their outside option. Further, in the pooling equilibria considered a buyer offers the fair share of the trade surplus and both types of seller get the same equilibrium utility. Given any out-of-equilibrium investment and the corresponding equilibrium reaction  $p \in \{0, \gamma\}$ , a selfish seller gets a weakly higher utility than a fair-minded seller. Therefore, a buyer facing some out-of-equilibrium investment can reasonably believe the seller to be selfish with high probability. Equilibrium refinements à la Intuitive Criterion have no bite.

Finally, intention-based fairness models naturally generate a discontinuity in the fair-minded sellers' reduced form preferences. Whether a buyer offers a fair-minded seller marginally less than the fair share or whether he claims the entire trade surplus, in both cases he expects the seller to reject. As in both cases a seller's material payoff then equals his outside option, the buyer is equally unkind. Thus, a sufficiently fair-minded seller retaliates marginal violations of the fairness norm. This causes the discontinuity in his reduced form preferences which can make signalling worthwhile.

## Appendix B: Proofs

The following lemma is useful for the ensuing formal analysis.

**Lemma 4** *For all  $\lambda \in [0, 1]$ , the function  $\lambda[\phi(i) - \psi(i)] + \psi(i) - i$  has a unique, positive, and finite maximizer. The corresponding maximum is positive. Both maximizer and maximum are strictly increasing in  $\lambda$ .*

### Proof of Lemma 4

Since  $\phi$  is strictly and  $\psi$  is weakly concave,  $\lambda[\phi(i) - \psi(i)] + \psi(i) - i$  is strictly concave. By the limit assumptions on  $\phi'$  it has a unique, strictly positive, and finite maximizer. Since  $\phi'(i) > \psi'(i)$  for all  $i$  and the implicit function theorem, the maximizer is strictly increasing

in  $\lambda$ . The corresponding maximum is positive. As  $\lambda' > \lambda$  implies  $\lambda' [\phi(i) - \psi(i)] + \psi(i) - i > \lambda [\phi(i) - \psi(i)] + \psi(i) - i$  for all strictly positive  $i$ , it is strictly increasing in  $\lambda$ . *Q.E.D.*

### Proof of Proposition 1

a) Consider a pooling equilibrium with equilibrium investment  $i^*$ . Since there is pooling the buyer holds belief  $\mu^*(i^*) = \pi$ . By Lemma 1 the buyer's most discouraging belief to any out-of-equilibrium investment  $i$  is  $\mu^*(i) = 0$ , thereby giving both types of sellers a deviation payoff  $\psi(i) - i$ . The highest deviation payoff is thus  $U_0$ . It is attainable if and only if  $i^* \neq i_0$ . As otherwise there exists a profitable deviation,  $U_\theta^* \geq U_0$  must hold for both  $\theta \in \{s, f\}$ . This condition is also sufficient since out-of-equilibrium beliefs can be chosen arbitrarily. Thus, there exists a pooling equilibrium if and only if  $U_\theta^* \geq U_0$ .

b) Suppose  $\pi \geq \gamma$ . By Lemma 1 the payoff for both types of sellers in a pooling equilibrium with investment  $i^*$  is  $\gamma[\phi(i^*) - \psi(i^*)] + \psi(i^*) - i^*$ . By part a) and the definition of  $I$  an investment  $i^*$  can thus be supported in equilibrium if and only if  $i^* \in I$ . As for all  $\gamma > 0$  the set  $I$  is non-empty by Lemma 2, such pooling equilibria exist.

c) Suppose  $\pi < \gamma$ . By Lemma 1 the payoff for both types of seller in a pooling equilibrium with investment  $i^*$  is  $\psi(i^*) - i^*$ . Only  $i^* = i_0$  satisfies  $U_\theta^* \geq U_0$  as required by part a) of this proof. Thus,  $i_0$  is the unique equilibrium pooling investment. *Q.E.D.*

### Proof of Proposition 2

a) Suppose there exists an equilibrium in which at least one type of a seller chooses an investment  $i^* \notin I$  with some strictly positive probability  $\sigma^*(i^*) > 0$ . By Lemma 1 the most favorable equilibrium proposal of the buyer is  $p_b^*(i^*) = \gamma$ . The seller optimally accepts and thus gets a payoff of  $\gamma[\phi(i^*) - \psi(i^*)] + \psi(i^*) - i^*$ . Because  $i^* \notin I$  this investment is strictly dominated by choosing  $i_0$  which generates  $U_0$ . Thus, there exists no PBE in which  $i^*$  is ever chosen with strictly positive probability.

b) Consider any investment  $i^* \in I$ . By Proposition 1 there exists a pooling equilibrium in which  $i^*$  is always chosen by both types if  $\pi \geq \gamma$ . This also holds true for  $\pi < \gamma$  if  $i^* = i_0$ . Otherwise, there exists a mixed-strategy equilibrium with the below properties in which  $i^*$  is chosen with strictly positive probability: First, consider the seller. The fair-minded seller accepts a proposal  $p$  if and only if  $p \geq \gamma$ . He always invests  $i^*$ . The selfish seller accepts a

proposal  $p$  if and only if  $p \geq 0$ . He invests  $i^*$  with probability  $\sigma_s^*(i^*) = [\pi(1 - \gamma)] / [\gamma(1 - \pi)]$ . Otherwise, he invests  $i_0$ . Note that  $\pi < \gamma$  implies  $\sigma_s^*(i^*) \in ]0, 1[$ . Second, consider the buyer. Observing  $i^*$  the buyer holds the belief  $\mu^*(i^*) = \gamma$ . He then proposes  $\gamma$  with probability

$$\sigma_b^*(i^*) = [U_0 - (\psi(i^*) - i^*)] / [\gamma[\phi(i^*) - \psi(i^*)]].$$

Otherwise, he proposes zero. Note that  $i^* \neq i_0$  and  $i^* \in I$  imply  $\sigma_b^*(i^*) \in ]0, 1[$ . Observing any investment  $i \neq i^*$  the buyer holds the belief  $\mu^*(i) = 0$ . He then proposes  $p_b^*(i) = 0$ .

c) Whenever possible the above beliefs are formed according to Bayes' rule. By Lemma 1 the buyer's and the sellers' strategies are mutually best responses given the beliefs. In particular,  $\sigma_b^*(i^*)$  is chosen such that the selfish seller is indifferent between investing  $i^*$  and  $i_0$ . Moreover,  $\sigma_s^*(i^*)$  is chosen such that the buyer holds belief  $\gamma$  when observing  $i^*$  and is thus indifferent between proposing  $\gamma$  and 0. *Q.E.D.*

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# Bargaining under Incomplete Information, Fairness, and the Hold-Up Problem \*

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## Abstract

In the hold-up problem incomplete contracts cause the proceeds of relationship-specific investments to be allocated by bargaining. This paper investigates the corresponding investment incentives if individuals have heterogeneous fairness preferences and thus differ in their bargaining behavior. Individual preferences are taken to be private information. Investments can then signal preferences and thereby influence beliefs and bargaining behavior. In consequence, individuals might choose high investments in order not to signal information that is unfavorable in the ensuing bargaining.

*JEL Classification: L2, C78, D82*

*Keywords: hold-up, relationship-specific investments, fairness, reciprocity, asymmetric information, signalling.*

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